Holography for Transport of non-Fermi Liquid

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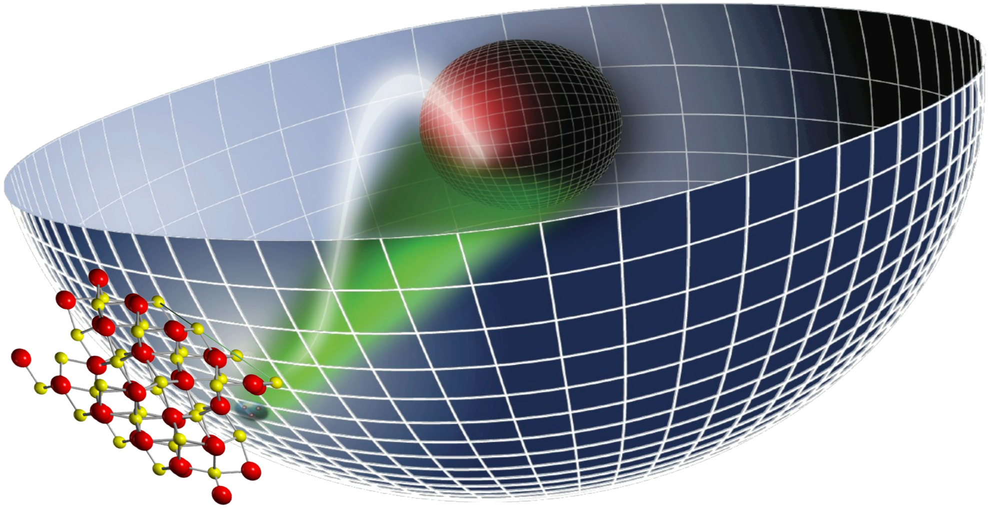
2016.05.29@USTC



Prologue

QCD, Strongly Correlated electron system and M-theory share a common fundamental problem :

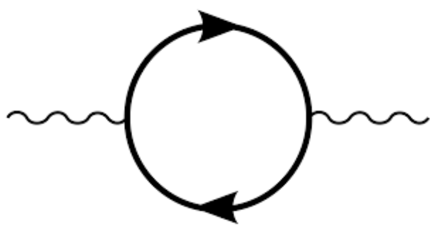
How to calculate the strongly interacting manybody system?

Interaction between particles

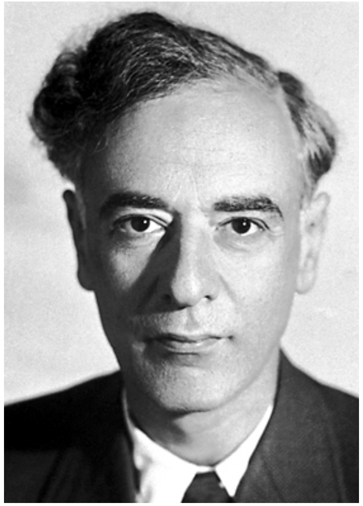
- If Neglected : manybody theory→ 1particle theory
  Band theory, shell models

- If weak : theory of quasi-particle = fermi liquid theory

$$\frac{1}{p^2 - m^2}$$

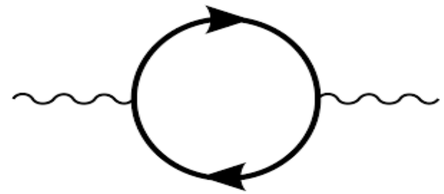


$$\frac{1}{p^2 - m_*^2 - \Sigma}$$



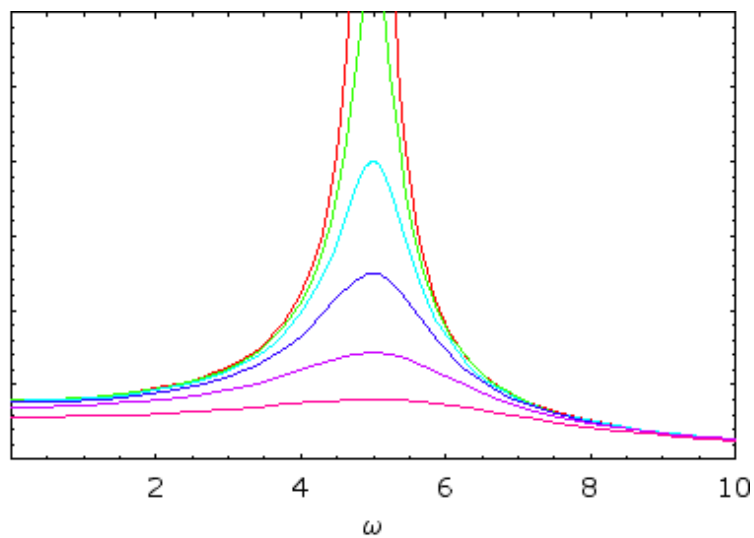
Effect of strong coupling

$$\frac{1}{p^2 - m^2}$$



$$\frac{1}{p^2 - m_*^2 - \Sigma}$$

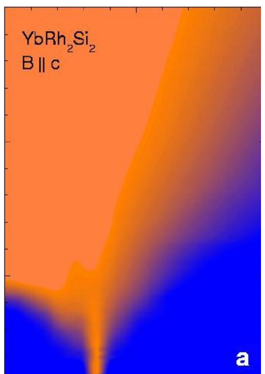
Look at the Spectral function as we increases the coupling:
Pole → resonance → background



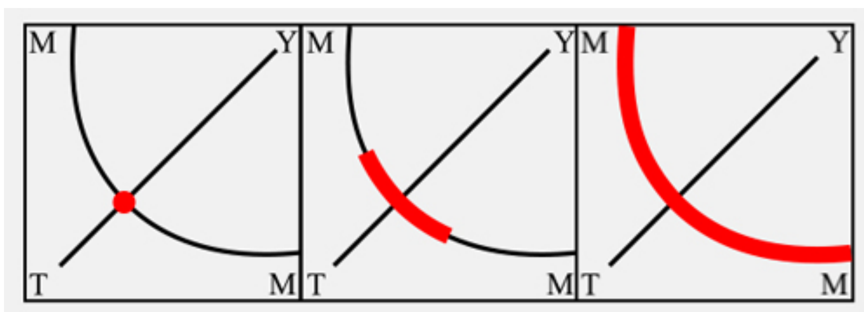
Strong coupling → Loss of particle character

After disappearance of (quasi-) particles?

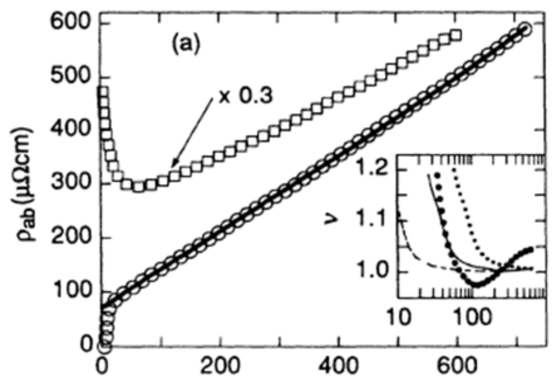
Quantum Critical Point, Fermi arc, Strange metal, …..
Characteristic phenomena of strongly interacting system➔ Non-Fermi Liquid



Quantum
Critical point



Fermi arc



T-linear Resistivity

Quantum Criticality

$$\mathcal{A} \sim c_0 + c_1 g + c_2 g^2 + \ldots$$

Divergence after regularization (at g_c)

- Meaning of divergence: Presence of phase transition

  If second order, QCP

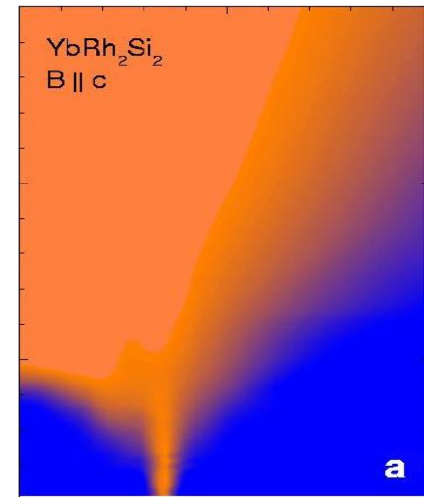
  1. loss of original degree of freedom
  2. appearance of new scale free field.

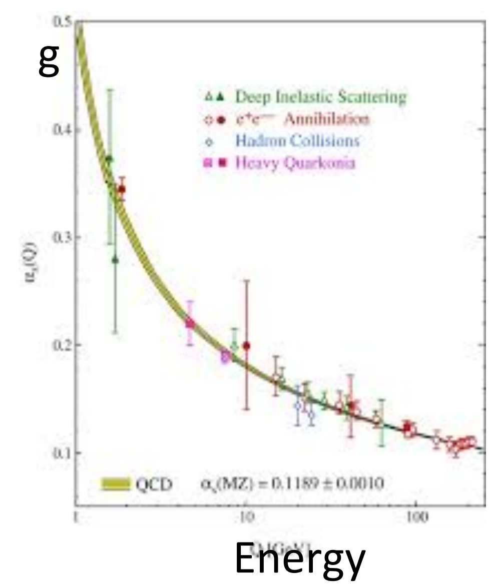
- Classification of QCP: Z, θ: $\omega = k^z$, $[s] = D - \theta$

- key: if massless particle has a relevant
- interaction → No solution in field theory

  ←→ QCD

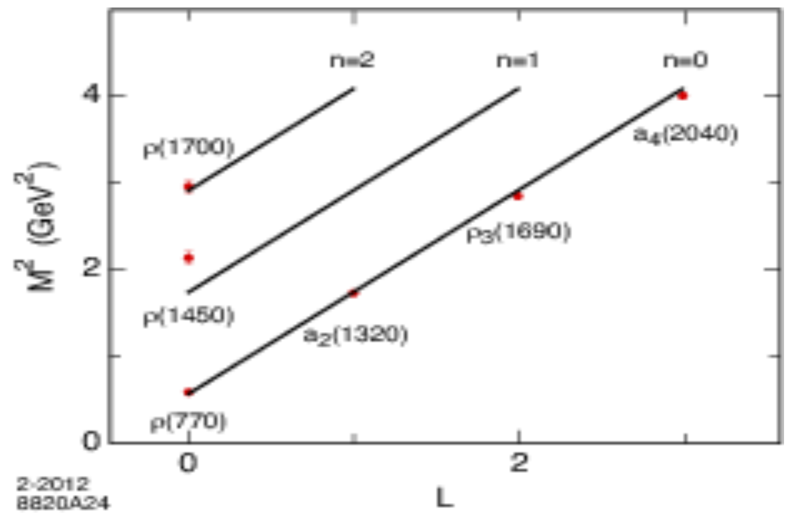
Need new field theory!





A Historical remark : birth of string Theory.

– 60’: 100 resonances.



– Gauge theory for isospin from 1954, but (str-coupling + mass prob)

– Field theory abandoned → S-matrix theory
Axiomatize: Analyticity, unitarity, relativity, crossing sym, causality,
(field theory property for weak coupling+ Bootstrap idea from here)

– 1968 Veneziano found a solution →Nambu, Nielson, Susskind read it as spectrum of vibrating string : appearance of extended object out of strong interaction! It fits data of Regge trajectory.

– MSG: String theory was discovered in solving strong coupling problem!

– And it was an alternative for low energy QCD.

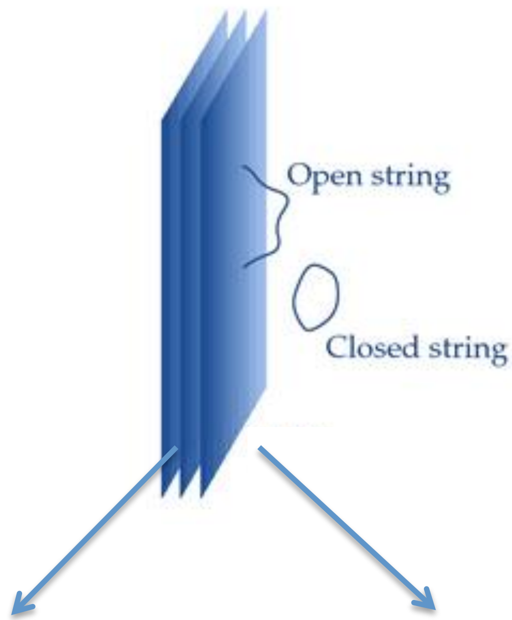
After 30 years

- Physicists came up with Another alternative :

AdS/CFT

Reminder

AdS/CFT (Maldacena 1997)



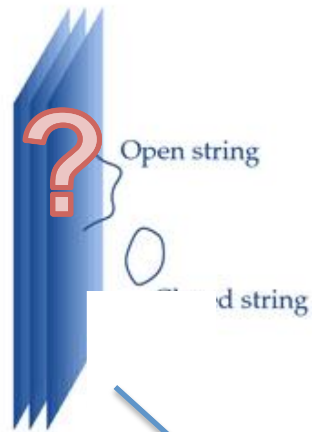
N=4 SYM

$AdS_5 \times S^5$

→ QCP(z,θ)

$$ds^2_{D+2} = r^{\frac{2\theta}{D}}\left(\frac{-dt^2}{r^{2z}} + \frac{dr^2}{r^2} + \frac{1}{r^2}\sum_{i=1}^{D} dx_i^2\right)$$

Def of AdS/CMT



$$ds^2_{D+2} = r^{\frac{2\theta}{D}}\left(\frac{-dt^2}{r^{2z}} + \frac{dr^2}{r^2} + \frac{1}{r^2}\sum_{i=1}^{D} dx_i^2\right)$$

QCP(z,θ)

Dual
?

Geometry (z,θ)

30 years of data

Holography model calculation

Transport coefficients

Brief history of holography

1997 discovery of AdS/CFT

1. 2002 Heavy Ion(QGP ~ perfect fluid) [Son et al.(Chicago)]
2. 2007 first ads/CMT paper by Sachdev (Harvard)
3. 2008 Superconductivity:Horowitz (UCSB); Gubser (Princeton)
4. 2008 Non-fermi liquid : Sung-Sik Lee (McMaster)
5. 2013 Metal-Insulator transition [Hartnoll (Stanford)]

……

6. 2014 Holography as a method of experimental science

Donos, Hartnoll, Gaunttlet, Tong, ….

calculational method of transport (still for z=1)

→ theory vs. exp possible

Optical conductivities for multi-order parameter

KY.Kim,KK.Kim,YS.Seo,Sin(1409.8346)

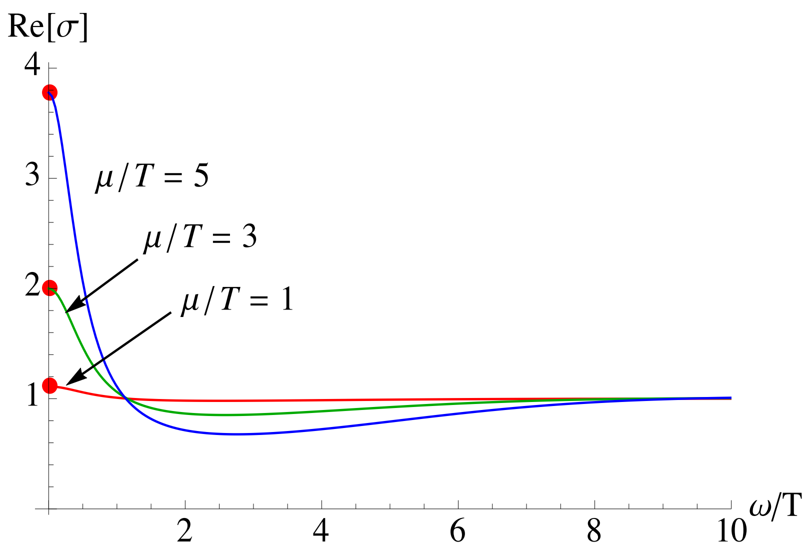
$$\begin{pmatrix} \langle J_x \rangle \\ \langle Q_x \rangle \end{pmatrix} = \begin{pmatrix} \sigma & \alpha T \\ \bar{\alpha} T & \bar{\kappa} T \end{pmatrix} \begin{pmatrix} E_x \\ -(\nabla_x T)/T \end{pmatrix}$$

The holographically renormalised action(S_{ren}) is given by

$$S_{\mathrm{ren}} = S_{\mathrm{EM}} + S_\psi + S_{\mathrm{c}}\,,$$

$$S_{\mathrm{EM}} = \int_M \mathrm{d}^4x \sqrt{-g}\left[R - 2\Lambda - \frac{1}{4}F^2\right] - 2\int_{\partial M} \mathrm{d}^3x\sqrt{-\gamma}K\,,$$

$$S_\psi = \int_M \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}\sum_{I=1}^{2}(\partial\psi_I)^2\right]\,,$$



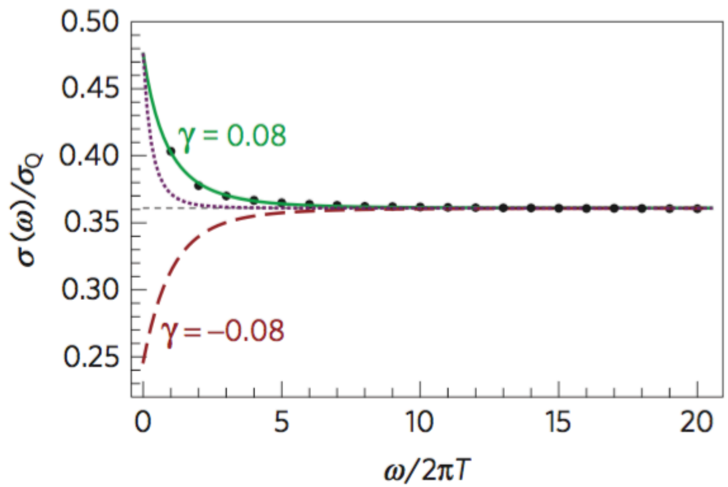
(a) Re σ

new era : data vs Holography : Quantum Monte Carlo

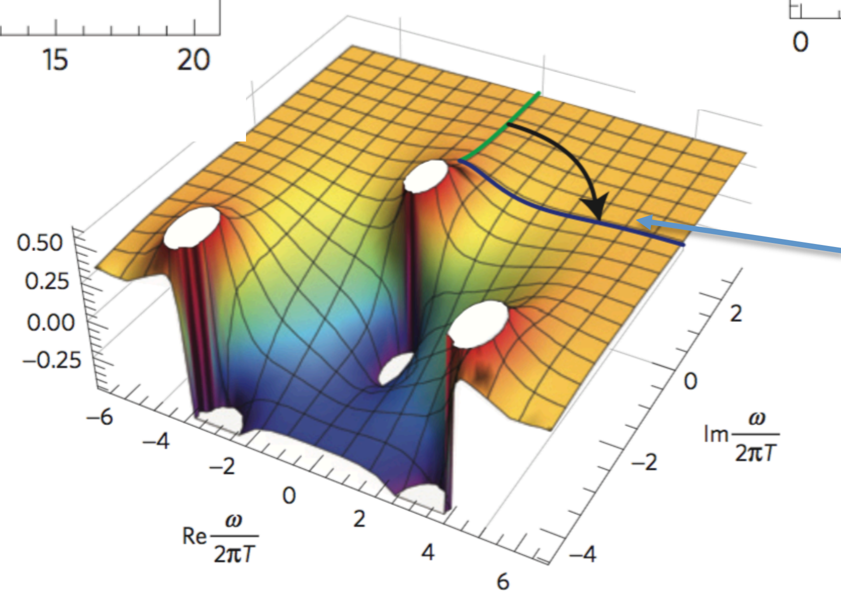
Sachdev *et al.* (Harvard), Nature Physics (2014)

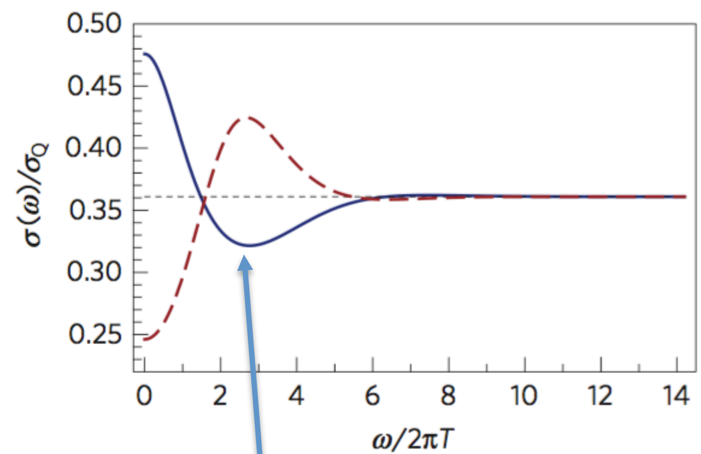
Bosonic Hubbard Model
superfluid-insulator QCP

$$S_{\rm bulk} = \int d^4x\sqrt{-g}\left(-\frac{1}{4g_4^2}F_{ab}F^{ab} + \gamma\frac{L^2}{g_4^2}C_{abcd}F^{ab}F^{cd}\right)$$



dots: Monte Carlo data
at imaginary frequencies

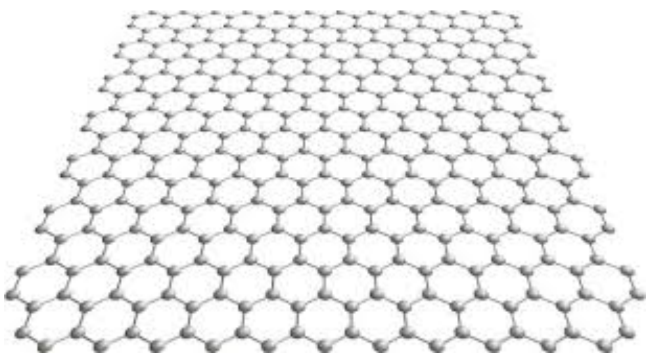


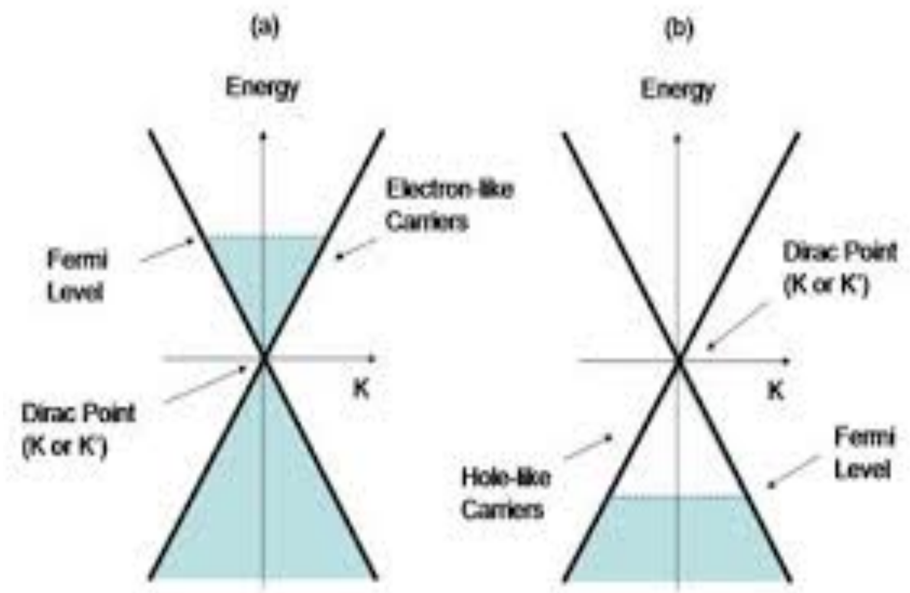


conductivity
at real frequencies

Holographic
continuation

Strong correlation in Graphene





Fermi level ~ near Dirac point : imperfect screening → strong correlation

→ Dirac Fluid (z=1)

2016: Experimental evidence of strong correlation

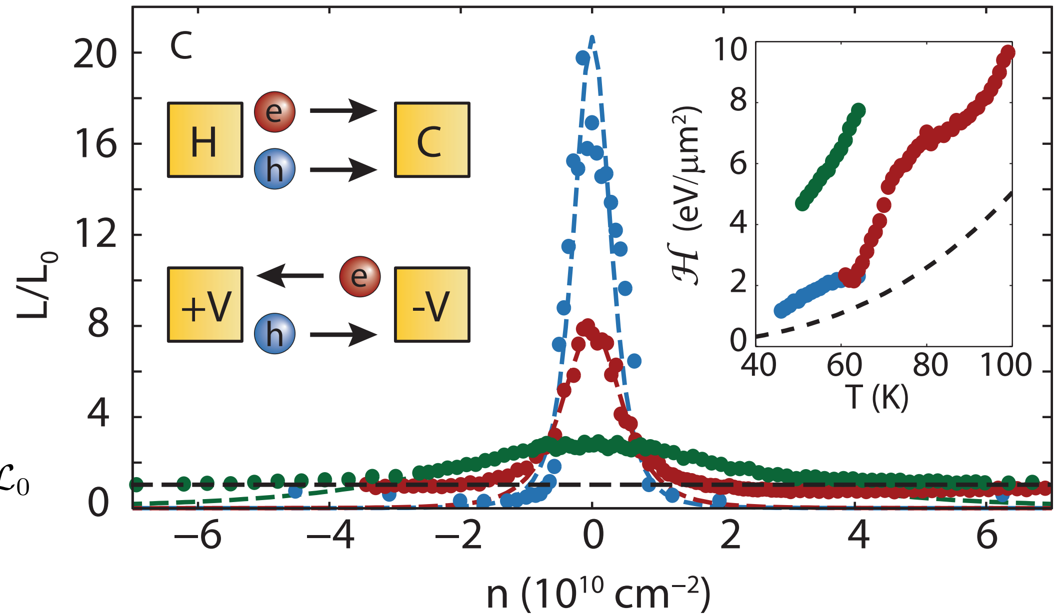
Observation of the Dirac fluid and the break-down of the Wiedemann-Franz law in graphene

Jesse Crossno, Jing . Shi, Ke Wang, Xiaomeng Liu, Achim Harzheim, Andrew Lucas, Subir Sachdev, Philip Kim,*, Takashi Taniguchi, Kenji Watanabe, Thomas A. Ohki5, Kin Chung Fong ,*

Science 11 Feb 2016

$$\mathcal{L} \equiv \frac{\kappa_{\mathrm{e}}}{\sigma T}$$

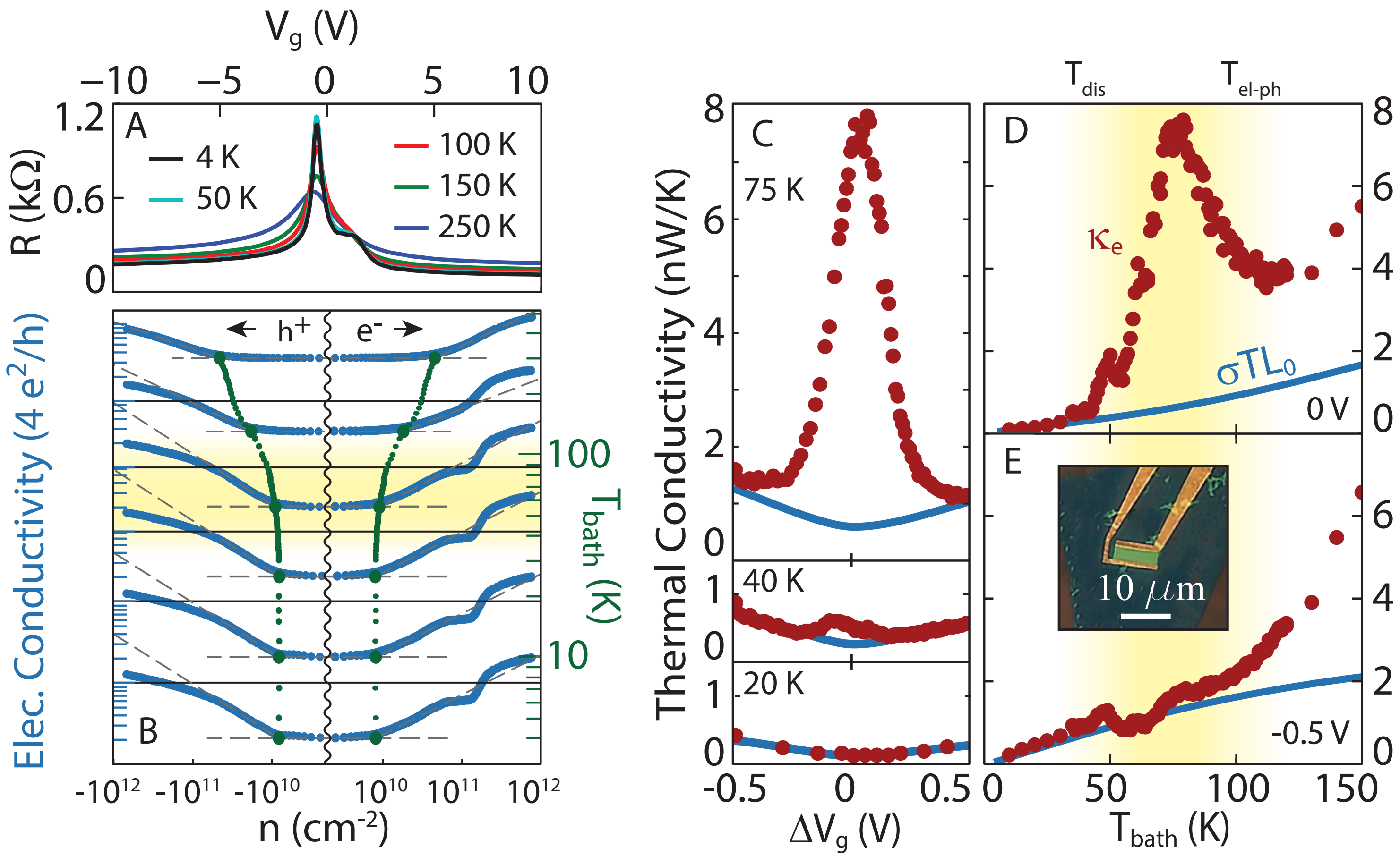
$$\frac{\pi^2}{3}\left(\frac{k_{\mathrm{B}}}{e}\right)^2 \equiv \mathcal{L}_0$$



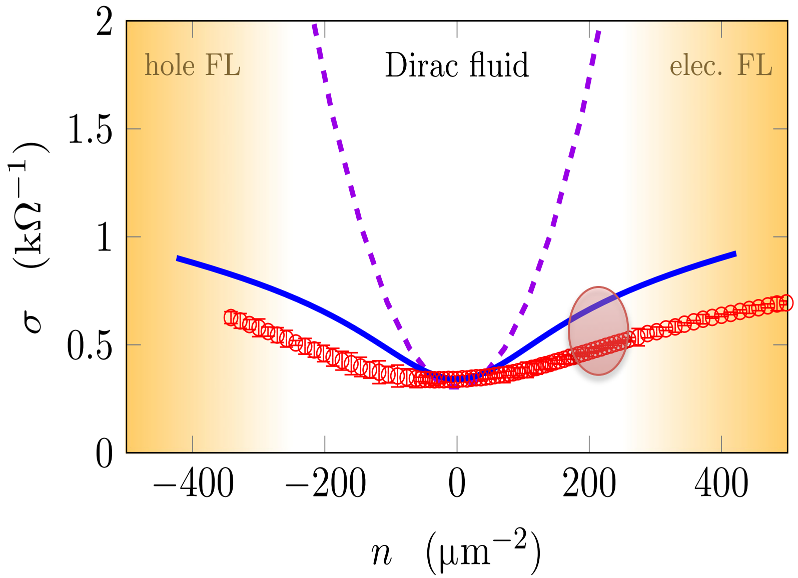
Continueed

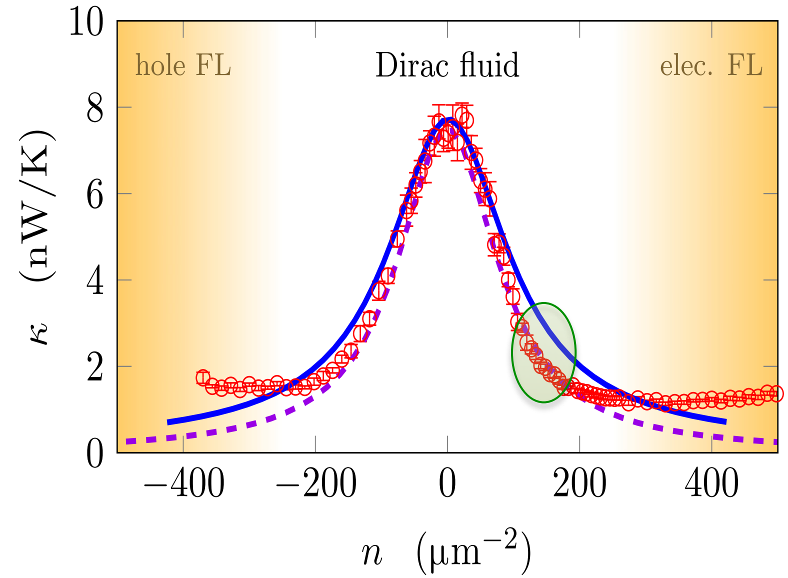
$$\mathcal{L} \equiv \frac{\kappa_e}{\sigma T}$$

~20, 7 by κ 1/3 by σ, around 60K



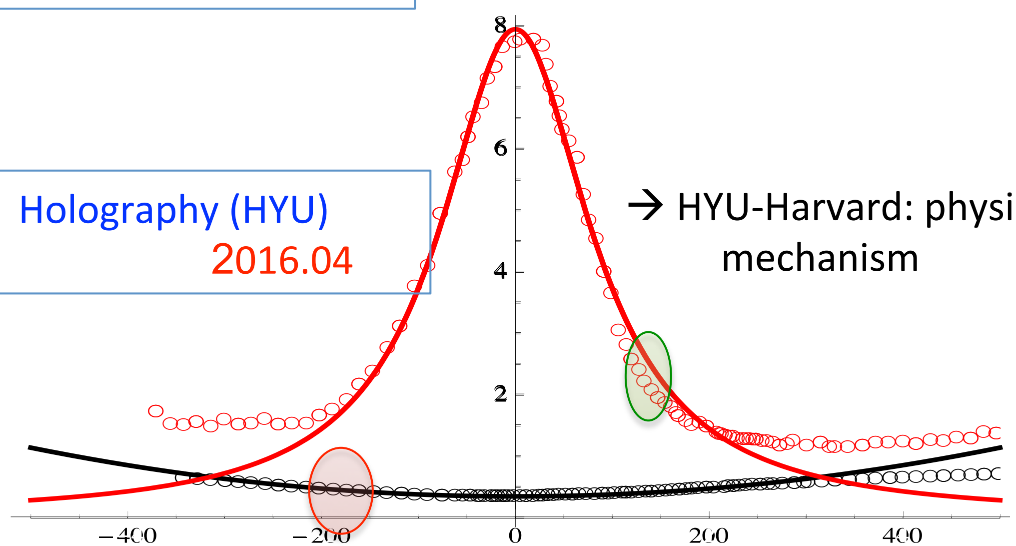
Theory vs. experiment





Fluid model(Sachdev group)

Holography (HYU)
2016.04



→ HYU-Harvard: physical mechanism

Two U(1) model

$$S = \int d^4x\sqrt{-g}\left[R - \frac{1}{2}\left[(\partial\phi)^2 + \Phi_1(\phi)(\partial\chi_1)^2 + \Phi_2(\phi)(\partial\chi_2)^2\right] - V(\phi) - \frac{Z(\phi)}{4}F^2 - \frac{W(\phi)}{4}G^2\right]$$

$$J_1 = \sqrt{-g}Z(\phi)F^{xr}$$

$$J_2 = \sqrt{-g}W(\phi)G^{xr}$$

$$Q = U(r)^2\frac{d}{dr}\left(\frac{\delta g_{tx}(r)}{U(r)}\right) - A(r)J_1,$$

$$\begin{pmatrix} \langle Q\rangle \\ \langle J_1\rangle \\ \langle J_2\rangle \end{pmatrix} = \begin{pmatrix} \bar{\kappa}T & \bar{\alpha}T & \bar{\gamma}T \\ \alpha T & \sigma & \bar{\delta} \\ \gamma T & \delta & \sigma_2 \end{pmatrix}\begin{pmatrix} -(\nabla_x T)/T \\ E \\ E_2 \end{pmatrix}$$

Transport of two U(1) model

$$\Sigma = \begin{pmatrix} \frac{(4\pi T)^2 e^{v_0}}{k^2 \Phi_0} & \frac{4\pi Q_1 T}{k^2 \Phi_0} & \frac{4\pi Q_2 T}{k^2 \Phi_0} \\ \frac{4\pi Q_1 T}{k^2 \Phi_0} & Z_0 + \frac{Q_1^2}{k^2 e^{v_0} \Phi_0} & \frac{Q_1 Q_2}{k^2 e^{v_0} \Phi_0} \\ \frac{4\pi Q_2 T}{k^2 \Phi_0} & \frac{Q_1 Q_2}{k^2 e^{v_0} \Phi_0} & W_0 + \frac{Q_2^2}{k^2 e^{v_0} \Phi_0} \end{pmatrix}$$

$$\sigma = Z_0 + \frac{\mu_1^2}{\beta^2}$$

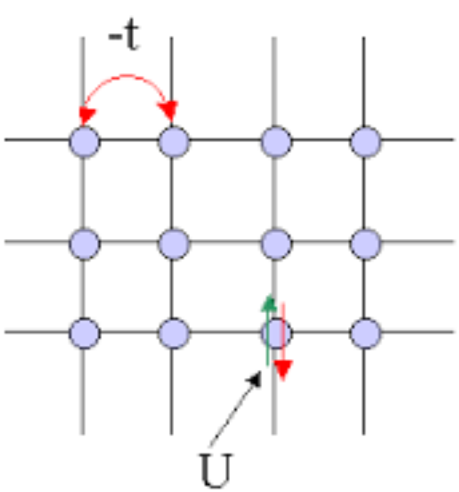
$$\kappa = \frac{(4\pi r_0)^2 \, T}{\beta^2 + \frac{\mu_1^2}{Z_0} + \frac{\mu_2^2}{W_0}}$$

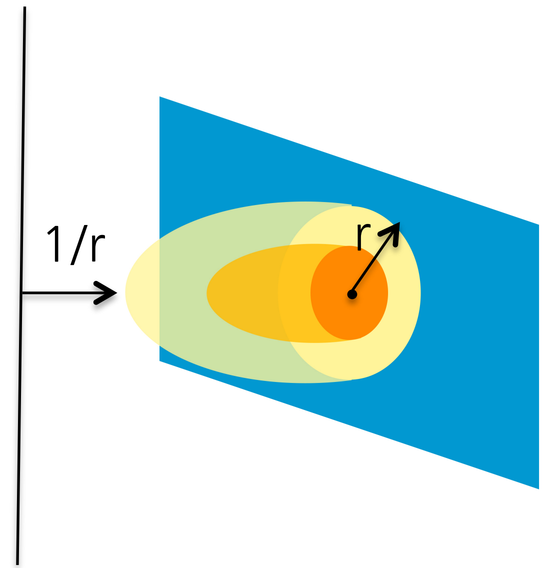
$$\mu_2 = g\mu_1$$

Why holography works in CMT (apart from universality)?

$$H = -t \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) - \mu \sum_i \hat{n}_i$$

$$S_{\text{bulk}} = \int d^4x \sqrt{-g} \left(-\frac{1}{4g_4^2} F_{ab}F^{ab} + \gamma \frac{L^2}{g_4^2} C_{abcd} F^{ab} F^{cd} \right)$$





Long ←→ Short

Expansion ←→ Falling

Repulsive Coulomb ←→ Attractive gravity

System characterization

- Holography is still in lack of one crucial elements : How to encode the system specific information?

- Two ways : i) Interaction in the bulk
  by Lifting least irrelevant interaction
  ii) lattice as boundary condition.

Lifting Least irrelevant int.

- ee int is already included by gravity but
  e-lattice should be encoded explicitly.

- 1. Integrate out massive fermion in 2+1 → Chern-Simon
  2. Lift it to the bulk → F^F
  3. Axionize for dynamical effect
     time reversal invariant case → $d\chi \wedge A \wedge F$
     time reversal Sym broken case → $(\partial\chi)^2 F \wedge F$

- Focus on T-broken case.
  Magnetization/Anomalous Hall effect /Pseudo gap/

Y.SEO, KY.Kim, KK.Kim, Sin arXiv:1512.08916

Model

$$S = \frac{1}{16\pi G}\int_{\mathcal{M}} d^4x\sqrt{-g}\left\{R + \frac{6}{L^2} - \frac{1}{4}F^2 - \frac{1}{2}(\partial\chi_I)^2\right\} + \frac{q_\chi}{16\pi G}\int_{\mathcal{M}} \frac{(\partial\chi_I)^2}{16} F\wedge F$$
$$- \frac{1}{16\pi G}\int_{\partial\mathcal{M}} d^3x\sqrt{-\gamma}\left(2K + \frac{4}{L} + R[\gamma] - \frac{L}{2}\nabla\chi_I\cdot\nabla\chi_I\right)$$

- Chern-Simon Axion term (with TRS broken) can represent mag neto-electric phenomena
- Dopping controls magnetic property as well as charge transpo rt.

Background solution (exact!)

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + r^2(dx^2 + dy^2)$$

$$A = a(r)dt + \frac{1}{2}H\,(xdy - ydx)$$

$$\chi_I = \beta\,\delta_{Ii}\,x^i.$$

$$U(r) = r^2 - \frac{\beta^2}{2} - \frac{m_0}{r} + \frac{q^2 + H^2}{4r^2} + \frac{\beta^4 H^2 q_\chi^2}{20r^6} + \frac{8\beta^2 H q q_\chi}{6r^4},$$

$$a(r) = \mu - \frac{q}{r} - \frac{\beta^2 H q_\chi}{3r^3}.$$

$$q = r_0\mu - \frac{\beta^2 H q_\chi}{3r_0^2}$$

$$m_0 = r_0^3\left(1 + \frac{r_0^2\mu^2 + H^2}{4r_0^4} - \frac{\beta^2}{2r_0^2}\right) + \frac{\beta^4 H^2 q_\chi^2}{45r_0^5}.$$

Fluctuation dynamics

- Fluctuations

$$\delta g_{tx}(t,r)=\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}e^{-i\omega t}\frac{r^2}{r_0^2}h_{tx}(\omega,r),\qquad \delta A_x(t,r)=\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}e^{-i\omega t}a_x(\omega,r)$$

$$\delta\psi_1(t,r)=\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}e^{-i\omega t}\chi(\omega,r)\,,$$

- EOMs in $h_{rx}=0\quad\text{gauge}$

$$\frac{\beta^2 h_{tx}}{r^2 f}+\frac{i r_0^2\beta\omega\chi}{r^2 f}-\frac{\mu r_0^3 a_x'}{r^4}-\frac{4h_{tx}'}{r}-h_{tx}''=0\,,$$

$$\frac{i\beta r_0^2 f\chi'}{r^2\omega}+\frac{\mu r_0^3 a_x}{r^4}+h_{tx}'=0\,,$$

$$\frac{f'a_x'}{f}+\frac{\mu h_{tx}'}{r_0 f}+\frac{\omega^2 a_x}{f^2}+a_x''=0\,,$$

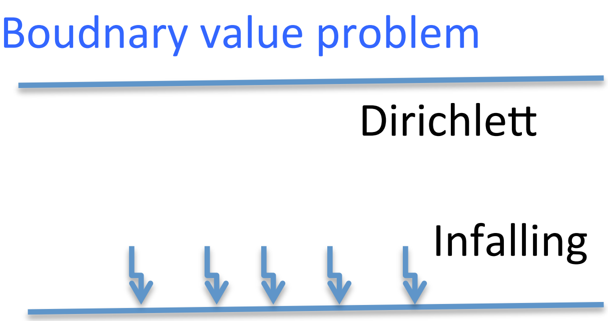
$$\frac{f'\chi'}{f}-\frac{i\beta\omega h_{tx}}{r_0^2 f^2}+\frac{\omega^2\chi}{f^2}+\frac{2\chi'}{r}+\chi''=0\,.$$

Independent

- Boundary action

$$S_{\text{ren}}^{(2)}=\lim_{r\to\infty}\frac{V_2}{2}\int \mathrm{d}\omega\left[-m_0\,h_{tx}h_{tx}-\mu\,a_xh_{tx}-f(r)a_xa_x'+r^4h_{tx}h_{tx}'-r^2f(r)\chi\chi'\right]$$

Boundary Condition



Determined by horizon regularity by the first three

$$h_{tx} = (r-1)^{\nu_\pm + 1}\left(h_{tx}^{(I)} + h_{tx}^{(II)}(r-1) + \cdots\right),$$

$$a_x = (r-1)^{\nu_\pm}\left(a_x^{(I)} + a_x^{(II)}(r-1) + \cdots\right),$$

$$\chi = (r-1)^{\nu_\pm}\left(\chi^{(I)} + \chi^{(II)}(r-1) + \cdots\right),$$

$$\nu_\pm = \pm i4\omega/(-12 + 2\beta^2 + \mu^2) = \mp i\omega/(4\pi T)$$

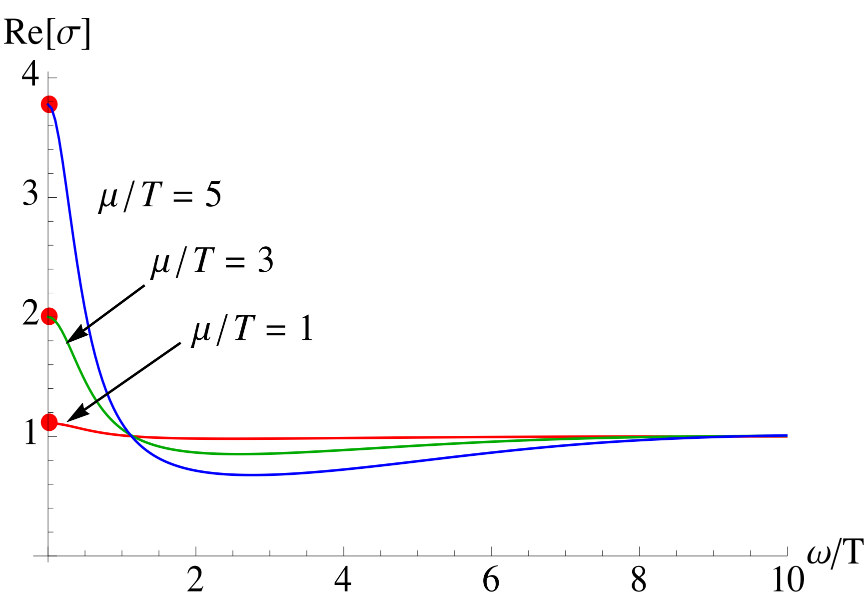
AC (electric,thermoelectric,thermal) conductivities

Kim,Kim,Seo,Sin(1409.8346)

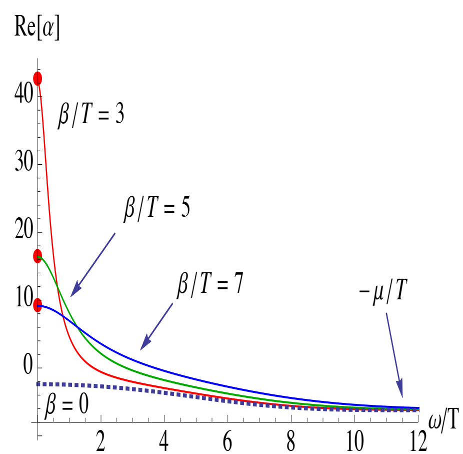
$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau} + \sigma_Q \, .$$

$$\sigma_Q = \left(\frac{3 - \frac{\mu^2}{4r_0^2}}{3 + \frac{3\mu^2}{4r_0^2}} \right)^2 , \quad K = r_0 \frac{\frac{\mu^2}{r_0^2}}{3 + \frac{3\mu^2}{4r_0^2}} ,$$

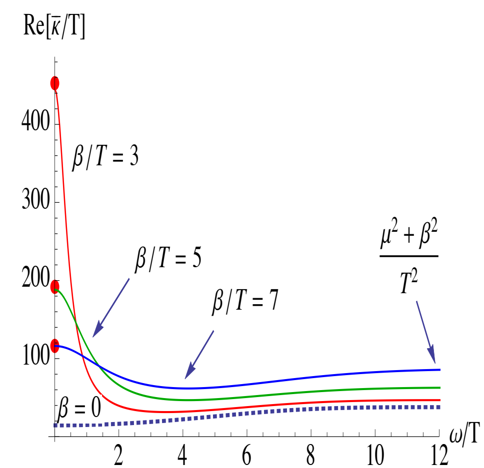
$$\begin{pmatrix} \langle J_x \rangle \\ \langle Q_x \rangle \end{pmatrix} = \begin{pmatrix} \sigma & \alpha T \\ \bar{\alpha} T & \bar{\kappa} T \end{pmatrix} \begin{pmatrix} E_x \\ -(\nabla_x T)/T \end{pmatrix}$$



(a) Re σ



(a) Re α



(b) Re $\bar{\kappa}/T$

Exact DC transports

$$\sigma_{xx} = \frac{(\mathcal{F} - H^2)(\mathcal{F} + \mathcal{G}^2)}{(\mathcal{F}^2 + H^2\mathcal{G}^2)}$$

$$\sigma_{xy} = \frac{H\mathcal{G}(2\mathcal{F} + \mathcal{G}^2 - H^2) + \theta(\mathcal{F}^2 + H^2\mathcal{G}^2)}{(\mathcal{F}^2 + H^2\mathcal{G}^2)}$$

$$\alpha_{xx} = \frac{s\mathcal{G}(\mathcal{F} - H^2)}{\mathcal{F}^2 + H^2\mathcal{G}^2}, \quad \alpha_{xy} = \frac{sH(\mathcal{F} + \mathcal{G}^2)}{\mathcal{F}^2 + H^2\mathcal{G}^2},$$

$$\mathcal{F} = r_0^2\beta^2 + H^2\left(1 - 14\theta^2/3\right) + 16\theta q H$$

$$\mathcal{G} = q - \theta H, \quad s = 4\pi r_0^2.$$

Anomalous Hall coefficient Rs

$$\rho_H \equiv R_H H + R_s M_0,$$

Experimental result is shown in right figure

$R_s \sim \rho_{xx}^2.$ For Intrinsic deflection / Side jump

$R_s \sim \rho_{xx}.$ For skew scattering

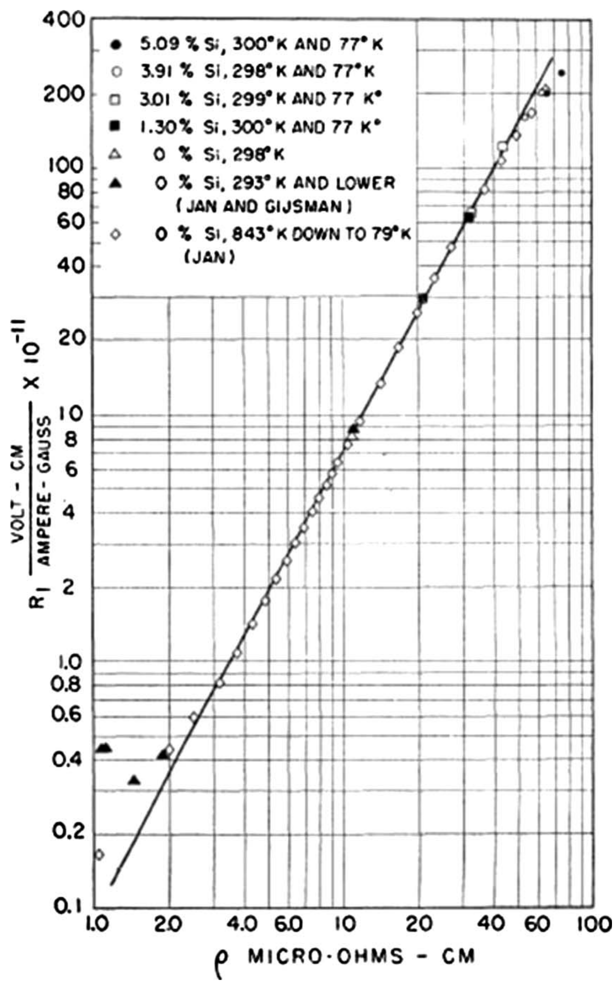


FIG. 2. Extraordinary Hall constant as a function of resistivity. The shown fit has the relation $R_s \sim \rho^{1.9}$. From Kooi, 1954.

Known Theory for anmalous Hall effect (Nagaosa et.al, Rev. Mod. Phys., Vol. 82, No. 2, April–June 2010)

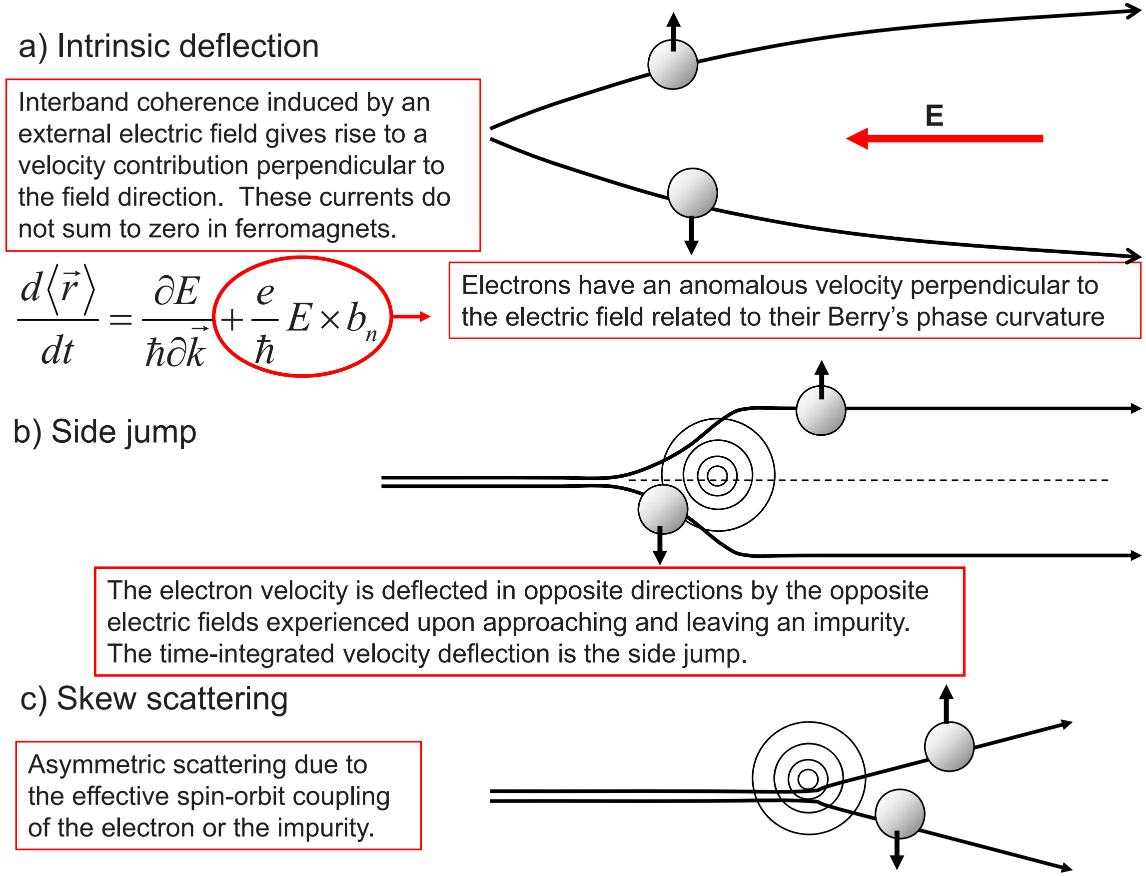
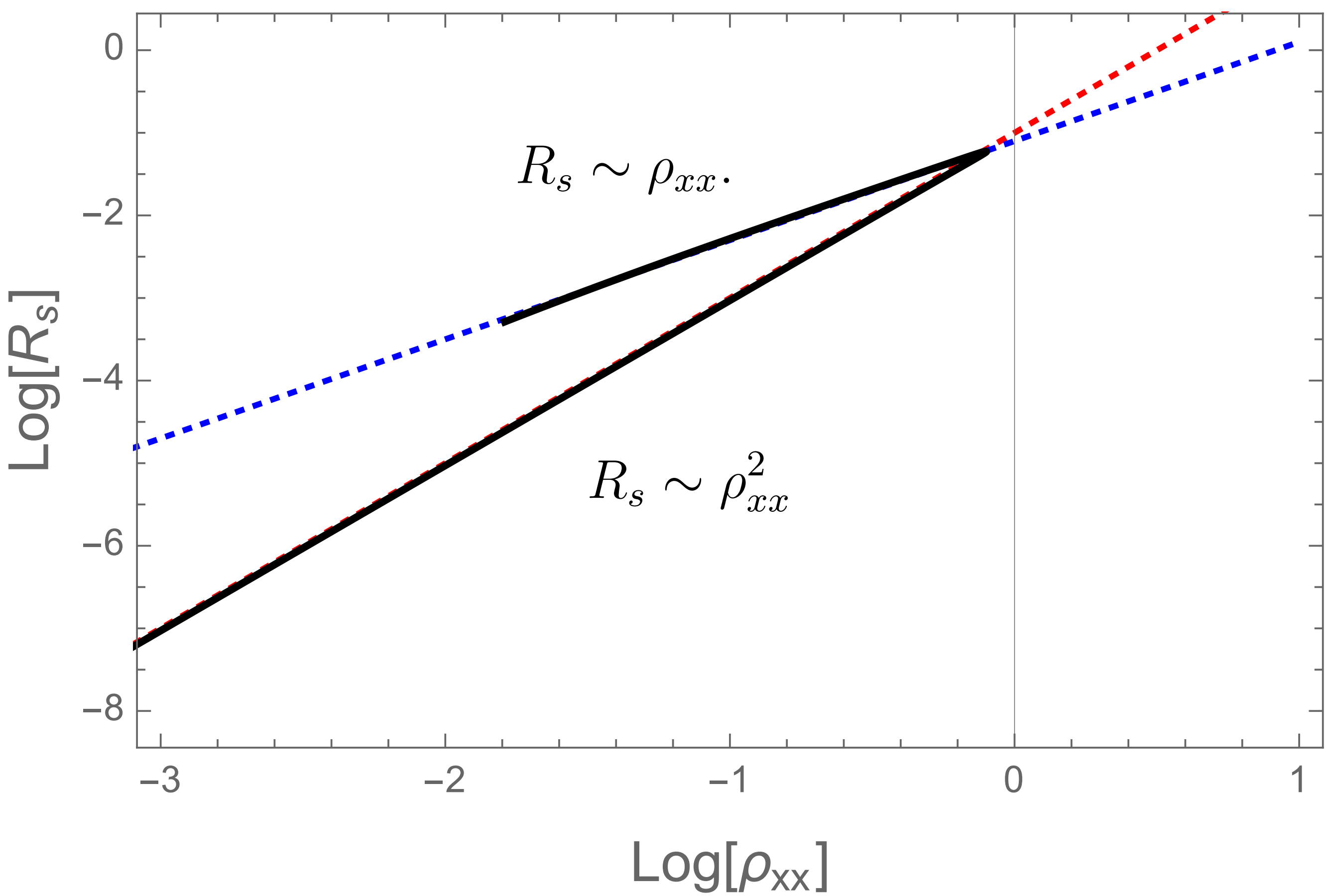


FIG. 3. (Color online) Illustration of the three main mechanisms that can give rise to an AHE. In any real material all of these mechanisms act to influence electron motion.

Our results from holography

$$R_s = \frac{3}{r_0\mu}\frac{1}{\theta^2 + (1+\mu^2/\beta^2)^2} \qquad \rho_{xx} = \frac{1+\mu^2/\beta^2}{\theta^2 + (1+\mu^2/\beta^2)^2}$$



Sharper transition at higher T

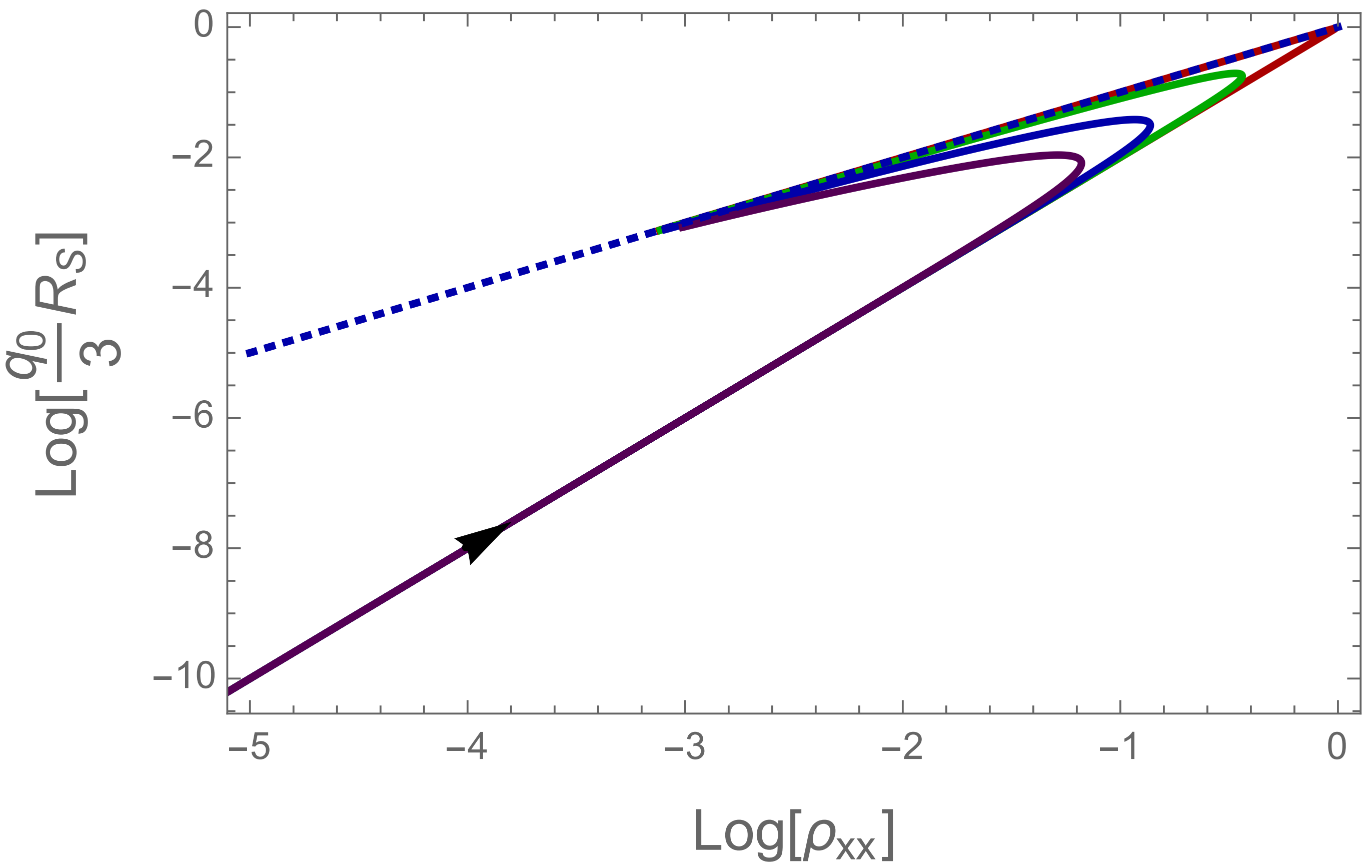


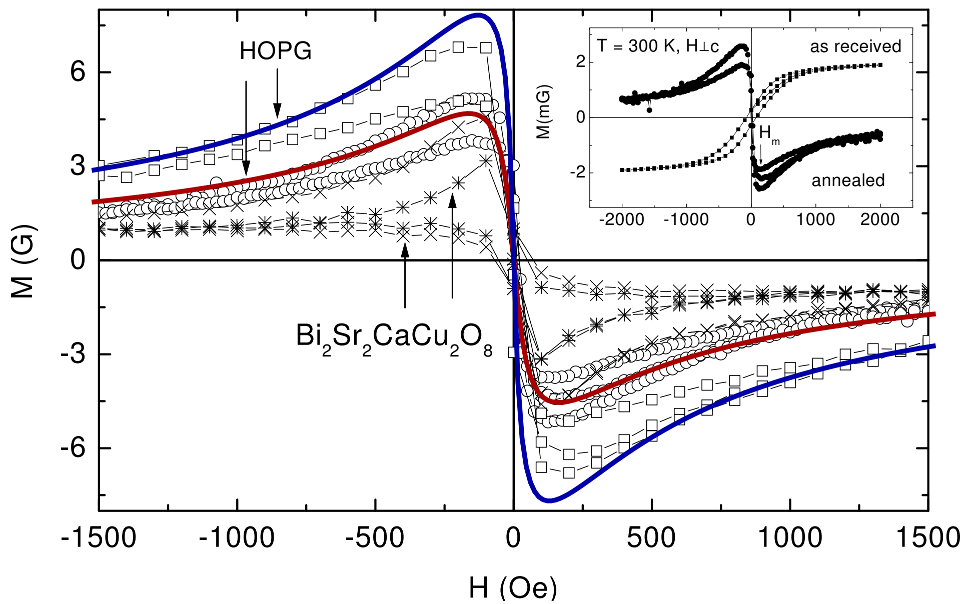
Figure 3: Relation between ρ_{xx} and $\frac{q_0}{3}R_s$ with $\mu/T = 0.1$(black), 1(red), 2(blue), 5(green) and 10(purple) in log-log plot. $\frac{q_0}{3}R_x \sim \rho_{xx}^2$ or $\sim \rho_{xx}$ in small (large) β/μ regime. Arrow direction is increasing β and we fix $q_\chi = 1$.

Diamagnetism in anealed graphite

SEO, KY.Kim, KK.Kim, Sin arXiv:1512.08916

$$2\kappa^2 S = \int d^4x\sqrt{-g}\left\{R + \frac{6}{L^2} - \frac{1}{4}F^2 - \sum_{I=1,2}\frac{1}{2}(\partial\chi_I)^2\right\} - \frac{1}{16}\int q_\chi(\partial\chi_I)^2 F\wedge F + S_c$$

$$M = -\frac{\partial F}{\partial H}\Big|_{fixed\ r_0,q,\beta} = \frac{1}{3}\theta q - \frac{1}{5}\theta^2 H, \quad \text{with } \theta = \frac{\beta^2 q_\chi}{r_0^2}.$$

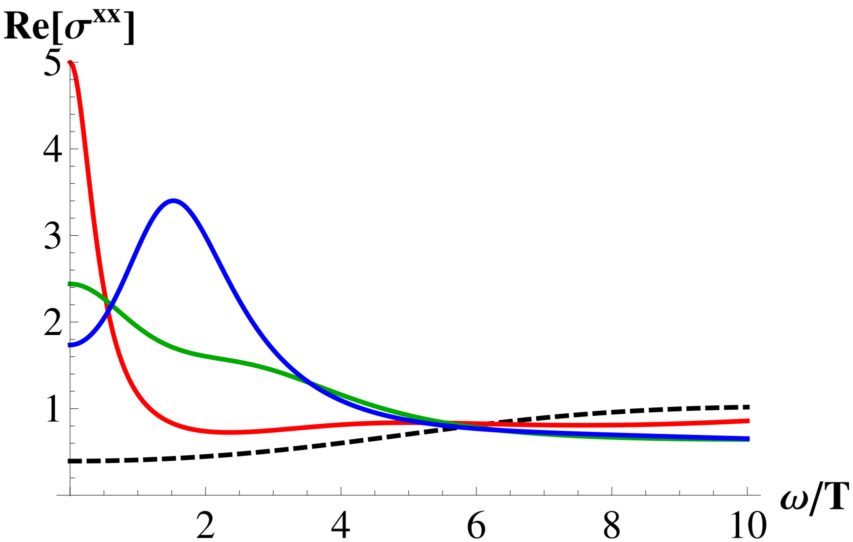


자화 곡선의
해석적 표현이 존재

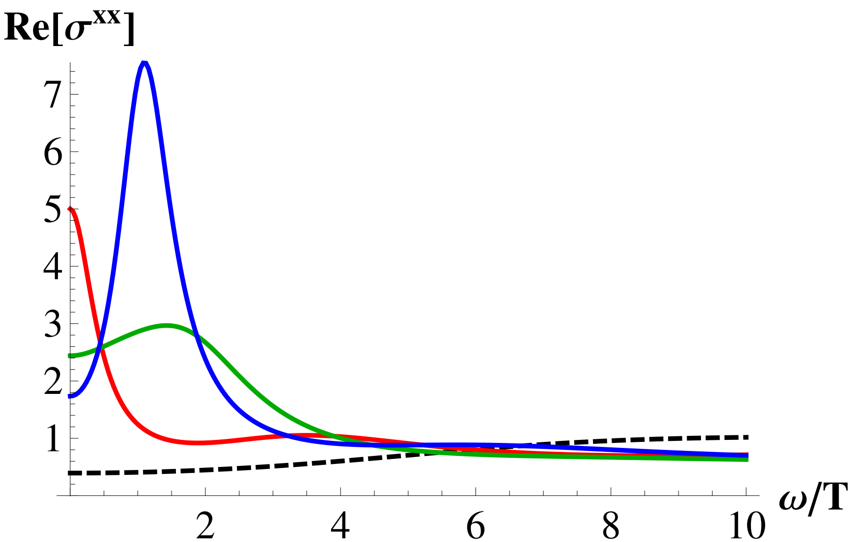
Data from

Y. Kopelevich, P. Esquinazi, J. H. S. Torres, and S. Moehlecke. Ferromagnetic- and superconducting-like behavior of graphite. *J. Low Temp. Phys.*, 119:691, 2000.

Pseudo Gap in optical conductivity



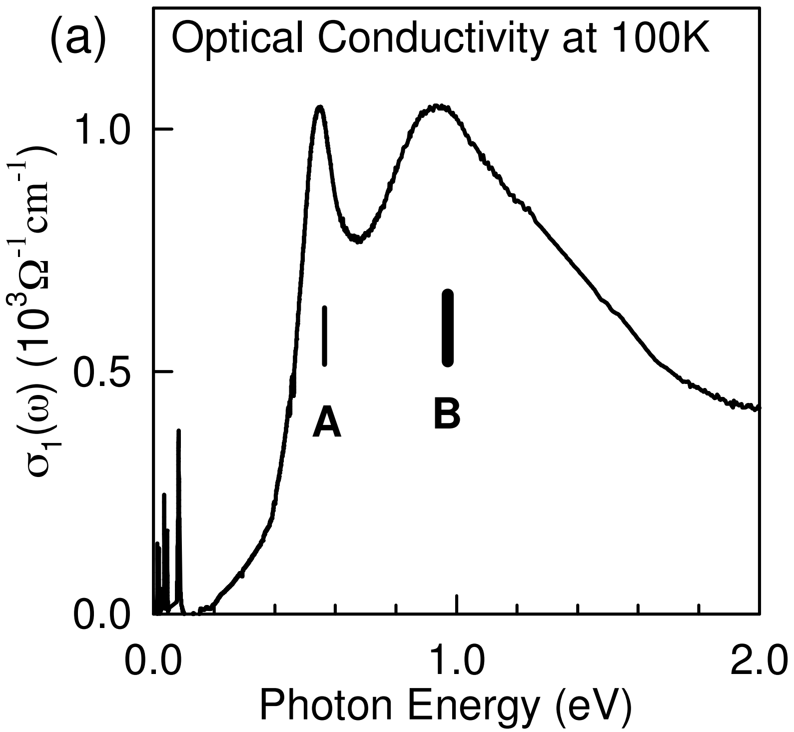
(a) $|q_\chi| = 3$



(a) $|q_\chi| = 5$

$$\beta/T \ = \ 0, 3, 5, 7 \ \text{(dashed, red, green, blue)}$$

Similarity? S-O induced Mott phenomena in Sr_2IrO_4



- B. J. Kim, Hosub Jin, S. J. Moon, J.-Y. Kim, B.-G. Park, C. S. Leem, Jaejun Yu, T. W. Noh, C. Kim, S.-J. Oh, J.-H. Park, V. Durairaj, G. Cao, and E. Rotenberg
- Phys. Rev. Lett. 101, 076402 – Published 15 August 2008

Conclusion

- A NEW field theory method for strongly correlated syst embased on holography is coming.

- To identify the system we need to identify key interactions in bulk local field theories.

- We considered magneto-electric term. The result is surprisingly good.

Thank you.